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NUMERICAL STUDIES OF SLOW VISCOUS ROTATING FLOW PAST A SPHERE—1

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SUMMARY

The Navier–Stokes equations for a steady, viscous rotating fluid, rotating about the z-axis with angular velocity ω are linearized using the Stokes approximation. The linearized Navier–Stokes equations governing the axisymmetric flow can be written as three coupled partial differential equations for the stream function, vorticity and rotational velocity components. One parameter, $R_{e\omega} = 2\omega a^2/\nu$, enters the resulting equations. For $R_{e\omega} \ll 1$, the coupled equations are solved by the Peaceman–Rachford A.D.I. (Alternating Direction Implicit) method and the resulting algebraic equations are solved by the 'method of sweeps'. Stream lines for $\psi = 0{\cdot}05$, $0{\cdot}2$, $0{\cdot}5$ and magnitude of the vorticity vector $z = 0{\cdot}2$ are plotted for $R_{e\omega} = 0{\cdot}1$, $0{\cdot}3$, $0{\cdot}5$. Correction to the Stokes drag due to the rotation of fluid is calculated.

1. INTRODUCTION

The Navier–Stokes equations are non-linear. The problem of axisymmetric flow past a sphere could not be solved analytically except by methods which first linearize the equations. Stokes[5] linearized them by neglecting the intertia terms in comparison with viscous terms and solved the equations. The Stokes solution is not uniformly valid throughout the flow field. Whitehead[8] tried to iterate on the Stokes solution and found that the condition at infinity could not be satisfied. Oseen[4] gave a solution, taking into consideration inertial terms, linearly where the condition on the body is satisfied approximately. Oseen's solution of linearized equations has been improved by Goldstein[2] and by Tomotika and Aoi[6].

The present problem consists of the governing differential equations which are the result of the Stokes type of linearization on the Navier–Stokes equations for the steady, incompressible, viscous rotating fluid, rotating about the z-axis with a uniform angular velocity ω. We obtain the three coupled equations when they are expressed in stream function ψ, vorticity ζ and rotational velocity component $\Omega = r \sin\theta V_\phi$; v_ϕ is the swirl velocity. The equations are solved numerically by the Alternating Direction Implicit (A.D.I.) method.[1,7,9]

2. FORMULATION OF THE PROBLEM

We consider the steady, slow, viscous rotating fluid which is in solid body rotation, with angular velocity ω about the z-axis and moving with a uniform velocity U along the z-direction. A sphere of radius a is introduced into the flow and kept fixed at the origin. We used spherical polar co-ordinates (r, θ, ϕ), and since the flow is axially symmetric about the z-axis, all quantities are independent of ϕ. The Navier–Stokes equations are

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$$\frac{1}{R_e}D^4\psi = \frac{2}{r^2\sin^2\theta}\left(\cos\theta\frac{\partial\Omega}{\partial r} - \frac{\sin\theta}{r}\frac{\partial\Omega}{\partial\theta}\right)$$

$$-\frac{1}{r^2\sin\theta}\frac{\partial(\psi, D^2\psi)}{\partial(r\theta)}$$

$$+\frac{2}{r^2\sin^2\theta}D^2\psi\left(\cos\theta\frac{\partial\psi}{\partial r} - \frac{\sin\theta}{r}\frac{\partial\psi}{\partial\theta}\right), \tag{1}$$

$$\frac{1}{R_e}D^2\Omega = -\frac{1}{r^2\sin\theta}\frac{\partial(\psi,\Omega)}{\partial(r,\theta)}, \tag{2}$$

where ,

$$D^2 = \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right),$$

and ψ is the stream function, Reynolds number $R_e = Ua/v$ is the kinematic viscosity, U is the uniform stream and Ω is the angular velocity. The above equations are written in non-dimensional form by taking

$$\frac{\psi'}{a^2U} = \psi, \quad \frac{r'}{a} = r, \quad \frac{v_t'}{U} = v_r, \quad \frac{v_\theta'}{U} = v_\theta,$$

$$\frac{v_\phi'}{U} = v_\phi, \quad c = \frac{2\omega a}{U}, \quad R_{e\omega} = \frac{2\omega a^2}{v},$$

where the primes denote dimensional quantities. The velocity components are given by

$$v_r = \frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta}, \quad v_\theta = -\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r}$$

and the swirl component

$$v_\phi = \frac{\Omega}{r\sin\theta}.$$

For slow motion and rotation the Navier–Stokes equations are linearized by taking v_r, v_θ, v_ϕ as $(v_r, v_\theta, v_\phi + \omega r\sin\theta)$ and neglecting squares and products or velocities. The linearized Navier–Stokes equations for the steady, viscous rotating fluid, rotating about the z-axis are the three coupled equations

$$D^2\psi = -r\sin\theta\zeta = -\zeta_1, \tag{3}$$

$$D^2\zeta_1 = R_{e\omega}\left(\cos\theta\frac{\partial\Omega}{\partial r} - \frac{\sin\theta}{r}\frac{\partial\Omega}{\partial\theta}\right), \tag{4}$$

$$D^2\Omega = -R_{e\omega}\left(\cos\theta\frac{\partial\psi}{\partial r} - \frac{\sin\theta}{r}\frac{\partial\psi}{\partial\theta}\right), \tag{5}$$

where ζ is the vorticity.

Equations (3), (4) and (5) are to be solved with the following boundary conditions.

$$\Omega = \psi = \frac{\partial\psi}{\partial r} = 0 \quad \text{on} \quad r = 1 \tag{6}$$

and

$$\left.\begin{aligned}\psi &= \tfrac{1}{2}r^2 \sin^2\theta \\ \Omega &= \frac{c}{2}r^2 \sin^2\theta\end{aligned}\right\} \text{ as } r \to \infty \tag{7}$$

for

$$\left.\begin{aligned}\theta = 0^\circ, \quad \psi = 0 \\ \theta = 180^\circ, \quad \psi = 0\end{aligned}\right\} \text{ axis of symmetry} \tag{8}$$

and for the vorticity

$$\zeta = -\frac{1}{r\sin\theta}D^2\psi,$$

the boundary conditions for

$$\left.\begin{aligned}\theta = 0^\circ, \quad \zeta = 0 \\ \theta = 180^\circ, \quad \zeta = 0\end{aligned}\right\} \text{ axis of symmetry} \tag{9}$$

$$\zeta \to 0 \quad \text{as} \quad r \to \infty. \tag{10}$$

The conditions for ζ at the surface of the sphere have to be determined from the condition of zero velocity at the surface, i.e. $\partial\psi/\partial r = 0$.

3. FINITE DIFFERENCE EQUATIONS

We assume that the sphere is situated on the axis of a cylindrical pipe of diameter approximately 7 times the sphere diameter and at the nearest lattice points to the pipe surface we assume the flow to be undisturbed and parallel. We choose boundary conditions at the surface of the containing pipe such that no flow through the pipe surface is ensured and that the velocity gradient at the surface is zero. This prevents the establishment of a parabolic velocity distribution in the pipe which would only confuse the results. These conditions are almost equivalent to the practical case of a sphere moving with uniform velocity along the axis of the pipe. We set $r = e^z$ in (2)–(10). The lattice points are the points of intersection of the circles $z =$ constant and $\theta =$ constant shown in Figure 1. The nodal points are $P(z_i, \theta_j), Q(z_{i+1}, \theta_j)$,

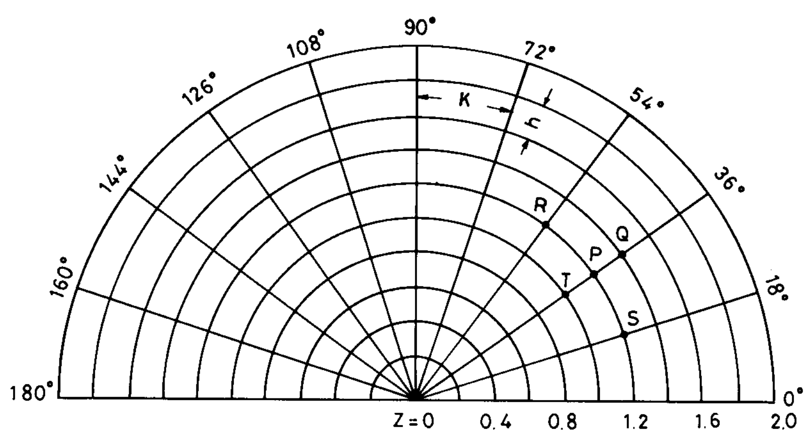

Figure 1. Finite difference mesh used for obtaining ψ

$R(z_i, \theta_{j+1}), S(z_i, \theta_{j-1})$ and $T(z_{i-1}, \theta_j)$. The length $z=0$ to $z=2$ is divided into 9 parts with step size $h=0{\cdot}2$, and $\theta=0$ to $\theta=\pi$ is divided into 9 parts with step length $k=10°$. We have in this lattice 81 nodal points. Finite difference equations of order h^2 and k^2 are written at each point for (3), (5) and the boundary conditions in (6)–(10). We denote at any point (z_i, θ_j) of ψ, ζ and Ω by $\psi_{i,j}, \zeta_{1_{i,j}}$ and $\Omega_{i,j}$. At each nodal point (z_i, θ_j) the finite difference equations (using central difference) are

$$-e^{2z_i}(\zeta_{1_{i,j}}) = \left(\frac{1}{h^2}-\frac{1}{2h}\right)\psi_{i+1,j} + \left(-\frac{2}{h^2}\right)\psi_{i,j} + \left(\frac{1}{h^2}+\frac{1}{2h}\right)\psi_{i-1,j} + \left(-\frac{\cot\theta_j}{2k}+\frac{1}{k^2}\right)\psi_{i,j+1} + \left(\frac{\cot\theta_j}{2k}+\frac{1}{k^2}\right)\psi_{i,j-1} - \frac{2}{k^2}\psi_{i,j}, \tag{11}$$

$$-R_{e\omega}e^{z_i}\left[\frac{\cos\theta_j}{2h}(\Omega_{i+1,j}-\Omega_{i-1,j}) - \frac{\sin\theta_j}{2k}(\Omega_{i,j+1}-\Omega_{i,j-1})\right] = \left(\frac{1}{h^2}-\frac{1}{2h}\right)\zeta_{1_{i+1,j}} - \frac{2}{h^2}\zeta_{1_{i,j}} + \left(\frac{1}{h^2}+\frac{1}{2h}\right)\zeta_{1_{i-1,j}} + \left(-\frac{\cot\theta_j}{2k}+\frac{1}{k^2}\right)\zeta_{1_{i,j+1}} - \frac{2}{k^2}\zeta_{1_{i,j}} + \left(\frac{\cot\theta_j}{2k}+\frac{1}{k^2}\right)\zeta_{1_{i,j-1}}, \tag{12}$$

$$-R_{e\omega}e^{z_i}\left[\frac{\cos\theta_j}{2h}(\psi_{i+1,j}-\psi_{i-1,j}) - \frac{\sin\theta_j}{2k}(\psi_{i,j+1}-\psi_{i,j-1})\right] = \left(\frac{1}{h^2}-\frac{1}{2h}\right)\Omega_{i+1,j} - \frac{2}{h^2}\Omega_{i,j} + \left(\frac{1}{h^2}+\frac{1}{2h}\right)\Omega_{i-1,j} + \left(\frac{-\cot\theta_j}{2k}+\frac{1}{k^2}\right)\Omega_{i,j+1} - \frac{2}{k^2}\Omega_{i,j} + \left(\frac{\cot\theta_y}{2k}+\frac{1}{k^2}\right)\Omega_{i,j-1}. \tag{13}$$

With the boundary conditions

$$\Omega_{0,j} = \psi_{0,j} = \left(\frac{\partial\psi}{\partial z}\right)_{0,j} = 0 \quad \text{on} \quad z=0 \tag{14}$$

on the container pipe $z=2$

$$\psi_{10,j} \sim \tfrac{1}{2}(e^4 \sin^2\theta_j)$$

$$\Omega_{10,j}\frac{c}{2}e^4\sin^2\theta_j \tag{15}$$

$$\zeta_{10,j} \to 0.$$

On the surface of the sphere, that is $z=0$,

$$\zeta_{0,j} = \left(\frac{8\psi_{1,j}-\psi_{2,j}}{2h^2\sin\theta_j}\right). \tag{16}$$

The three coupled equations (11) to (14) with the boundary conditions (14) to (16) are solved using the Peaceman–Rachford A.D.I. method. In this method, the equations (11)–(13) are written in two steps; they are as follows.

(i) Horizontal sweep (along $r = \text{constant}$).
(ii) Vertical sweep (along $\theta = \text{constant}$).

The well known A.D.I. and successive line over-relaxation (SLOR) methods are applied to elliptic equations. A brief resumé of the Peaceman–Rachford A.D.I. method which is applied to the problem solved is given here. The A.D.I. method applied here is a two-step method involving the solution of a tridiagonal set of equations along lines parallel to the x_1 and x_2 axes as the first and second steps, respectively. We consider the system of algebraic equations

$$Au = b \tag{17}$$

which is based on the splitting of the matrix A into the sum of three matrices

$$A = H_0 + V_0 + \Sigma, \tag{18}$$

where Σ is a non-negative diagonal matrix, and H_0, V_0 and Σ satisfy the following conditions:

(a) $H_0 + \theta\Sigma + \rho I$ and $U_0 + \theta\Sigma + \rho I$ are non-singular for any $\rho > 0, \theta \geqslant 0$;
(b) for any vectors c and d and for any constants $\rho > 0$ and $\theta \geqslant 0$, it is convenient to solve the system

$$(H_0 + \theta\Sigma + \rho I)x = c, \quad (V_0 + \theta\Sigma + \rho I) = d \tag{19}$$

for x and y respectively.

Here H_0 and V_0 are triangular matrices and H_0 corresponds to the term with derivatives in x, and U_0 corresponds to the term with derivatives in y. ρ and θ are accelerating parameters.
Let us write (17) in the forms

$$(H_0 + \theta\Sigma + \rho I)u = b - (V_0 + (1-\theta)\Sigma - \rho I)u, \tag{20}$$

$$(U_0 + \hat{\theta}\Sigma + \rho I)u = b - (H_0 + (I - \hat{\theta})\Sigma - (\rho' I)u. \tag{21}$$

In the Peaceman–Rachford method (1955) one selects positive iteration parameters ρ and ρ' and determines $u^{(n+1/2)}$ by

$$(H_0 + \theta\Sigma + \rho I)u^{(n+1/2)} = b - (V_0 + (1-\theta)\Sigma - \rho I)u^n. \tag{22}$$

Then one determines u^{n+1} by

$$(V_0 + \theta\Sigma + \rho I)u^{n+1} = b - (H_0 + (1-\theta)\Sigma - \rho I)u^{n+1/2}. \tag{23}$$

For simplicity, we consider here the special case where $\theta = \hat{\theta} = 1/2$, $\rho = \rho'$ and we let

$$H = H_0 + \tfrac{1}{2}\Sigma, \quad V = V_0 + \tfrac{1}{2}\Sigma. \tag{24}$$

H and V satisfy the following conditions:

(a) $H + \rho I$ and $V + \rho I$ are non-singular for any $\rho > 0$;
(b) for any vectors c and d and for any $\rho > 0$ it is convenient to solve the systems

$$(H + \rho I)x = c, \quad (V + \rho I)y = d. \tag{25}$$

Thus (17) becomes

$$(H + V)u = b \tag{26}$$

and (22) and (23) become, respectively,

$$(H+\rho I)u^{(n+1/2)} = b - (V - \rho I)u^{(n)}. \tag{27}$$

Horizontal sweep

$$(V+\rho I)u^{(n+1)} = b - (H - \rho I)u^{(n+1/2)}. \tag{28}$$

Vertical sweep

The determination of $u^{(n+1/2)}(x, y)$ involves the solution of a linear system with a tridiagonal matrix. This is also true for the determination of $u^{(n+1)}(x, y)$.

This split formula with $u^{(n+1/2)}$ and $u^{(n+1)}$ was introduced for the first time by Peaceman and Rachford and is therefore referred to in the literature as the Peaceman–Rachford formula. Since computation is done alternately in two directions and is implicit, it is called the A.D.I. method. The completion of these two steps constitutes a step forward in time.

Equation (11) is written in the form for horizontal sweep:

$$\left(\frac{1}{h^2}-\frac{1}{2h}\right)\psi_{i+1,j}-\frac{2}{h^2}\psi_{i,j}+\left(\frac{1}{h^2}+\frac{1}{h^2}\right)\psi_{i-1,j} = -e^{2z_i}\zeta_{1_{i,j}} \quad (i, j = 1, 2, \ldots, 9).$$

At any point (z_i, θ_j) we have

$$\left[\left(\frac{1}{h^2}-\frac{1}{2h}\right), -\frac{2}{h^2}, \left(\frac{1}{h^2}+\frac{1}{2h}\right)\right]\begin{bmatrix}\psi_{i+1,j}\\ \psi_{i,j}\\ \psi_{i-1,j}\end{bmatrix} = -e^{2z_i}\zeta_{1_{i,j}} \quad (i, j = 1, 2, \ldots, 9).$$

The matrix form for all the points is

$$[H][\psi] = b \tag{29}$$

where

$$[H] = \left[\left(\frac{1}{h^2}-\frac{1}{2h}\right), -\frac{2}{h^2}, \left(\frac{1}{h^2}+\frac{1}{2h}\right)\right]$$

and

$$b = -e^{2z_i}\zeta_{1_{i,j}},$$

$$[\psi] = \begin{bmatrix}\psi_{i+1,j}\\ \psi_{i,j}\\ \psi_{i-1,j}\end{bmatrix}$$

are unknowns.

For vertical sweep

$$\left(-\frac{\cot\theta_j}{2k}+\frac{1}{k^2}\right)\psi_{i,j+1}+\left(\frac{\cot\theta_j}{2k}+\frac{1}{k^2}\right)\psi_{i,j-1}-\frac{2}{k^2}\psi_{i,j} = b.$$

At any point (z_i, θ_j) we have

$$\left[\left(\frac{-\cot\theta_j}{2k}+\frac{1}{k^2}\right), -\frac{2}{k^2}, \left(\frac{\cot\theta_j}{2k}+\frac{1}{k^2}\right)\right]\begin{bmatrix}\psi_{i,j+1}\\ \psi_{i,j}\\ \psi_{i,j}\end{bmatrix} = b_i.$$

The matrix form for all the points is

$$[V][\psi]=[b] \tag{30}$$

where

$$[V]=\left[\left(\frac{-\cot\theta_j}{2k}+\frac{1}{k^2}\right),-\frac{2}{k^2},\left(-\frac{\cot\theta_j}{2k}+\frac{1}{k^2}\right)\right]$$

and

$$[\psi]=\begin{bmatrix}\psi_{i,j+1}\\ \psi_{i,j}\\ \psi_{i,j-1}\end{bmatrix}$$

are unknowns.

We write equations (29) and (30) in the following form:

$$(H-\rho I)\psi^{(n+1/2)}=b-(V-\rho I)\psi^{(n)}, \tag{31}$$

$$(V-\rho I)\psi^{(n+1)}=b-(V-\rho I)\psi^{(n+1/2)}, \tag{32}$$

where I is the unit matrix, ρ is the acceleration parameter, and H and V are horizontal and vertical matrices.

The $\psi^{(n)}$ are unknowns. Using initial values for $\psi^{(n)}$ and ζ which are zeros, we obtain $\psi^{(n+1/2)}$. Using $\psi^{(n+1/2)}$ in equation (32) we obtain $\psi^{(n+1)}$. Similarly we can write for equations (12) and (13). In this method, to ensure diagonal dominance, the acceleration parameter ρ is chosen as 130.

Iterations used for convergence

The number of iterations required to ensure convergence in the solution of equations (11)–(13) with the boundary conditions (14)–(16) are given in Table I.

Equation (13) had to be iterated ten times before two successive iterated values of Ω were coincident. These Ω values are used in equation (12) to obtain $\zeta_1 \cdot \zeta_1$ values are then used in equation (11) which is iterated ten times to obtain the ψ values. The process had to be repeated ten times before convergence was obtained. The starting values for ψ, ζ_1 and Ω are taken as zeros.

There is a good coincidence for horizontal and vertical sweeps at the point (z_i, θ_j), where $(i,j)=1,2,\ldots,9$.

A computer program for the Peaceman–Rachford A.D.I. method where lattice points are the points of intersection on $z=\text{constant}$ (circles) and $\theta=\text{constant}$ (lines) and not as vertices of rectangles has been developed on an IBM-370/155 compute. The resulting algebraic equations are solved by the method of sweeps. We divide the mesh into nine blocks, where each block is a tridiagonal system.

Table I.

Equation number	Number of iterations
(13)	10
(12)	0
(11)	10

4. RESULTS

For the set of values $R_{e\omega} = 0{\cdot}1, 0{\cdot}3, 0{\cdot}5$ the stream function ψ is given in Table II for the method of sweeps. The vorticity components ξ, η and ζ are given by

$$\xi = \frac{1}{r^2 \sin\theta}\frac{\partial\Omega}{\partial\theta}, \quad \eta = \frac{1}{r\sin\theta}\frac{\partial\Omega}{\partial r}, \quad -\zeta = \frac{1}{r^2\sin\theta}D^2\psi.$$

Hence the magnitude of the vorticity is equal to $\sqrt{\xi^2+\eta^2+\zeta^2}$. For the same values of $R_{e\omega}$, the magnitude of the vorticity vector is given in Table III for the method of sweeps. In Table IV the Stokes stream function is given so as to compare with the rotating case. The effect of rotation on the Stokes drag, D_s, is found as shown in Table V.

Table II. Solution $R_{e\omega} = 0{\cdot}1$: stream function

z/θ	18	54	90	126	162
0·2	0·004879	0·031714	0·047023	0·030108	0·004445
0·4	0·022604	0·147086	0·218541	0·140284	0·020754
0·6	0·060215	0·392453	0·584295	0·375839	0·055680
0·8	0·128278	0·837804	1·249890	0·805354	0·119397
1·0	0·242125	1·585768	2·370996	1·530107	0·226872
1·8	1·732000	11·671254	17·721466	11·546071	1·694576
		Solution $R_{e\omega} = 0{\cdot}3$: stream function			
0·2	0·005289	0·033733	0·048164	0·029243	0·004107
0·4	0·024335	0·155682	0·223549	0·136754	0·019340
0·6	0·064357	0·413275	0·596686	0·367287	0·058204
0·8	0·136066	0·877610	1·273873	0·788316	0·112470
1·0	0·254744	1·651677	2·410841	1·499534	0·214595
1·8	1·748600	11·775170	17·779053	11·459915	1·664309
		Solution $R_{e\omega} = 0{\cdot}5$: stream function			
0·2	0·005652	0·035911	0·050187	0·029099	0·003865
0·4	0·025862	0·164966	0·232416	0·136392	0·018346
0·6	0·067921	0·435549	0·618580	0·366566	0·049746
0·8	0·142488	0·919384	1·316137	0·786426	0·107419
1·0	0·264512	1·718857	2·480821	1·494087	0·205185
1·8	1·756028	11·861333	17·878387	11·405913	1·632364

Table III

18°	36°	54°	72°	90°	108°	126°	144°	162°
			Solution $R_{e\omega} = 0{\cdot}1$: magnitude of the vorticity vector at $z = 0{\cdot}2$					
0·780952	1·238541	1·618835	1·855525	1·921924	1·815028	1·551643	1·166812	0·725414
			Solution $R_{e\omega} = 0{\cdot}3$: magnitude of the vorticity vector at $z = 0{\cdot}2$					
0·841009	1·320202	1·704112	1·922181	1·953064	1·805706	1·510990	1·114305	0·681474
			Solution $R_{e\omega} = 0{\cdot}5$: magnitude of the vorticity vector at $z = 0{\cdot}2$					
0·902201	1·405162	1·798653	2·005910	2·007921	1·822920	1·494880	1·080447	0·648461

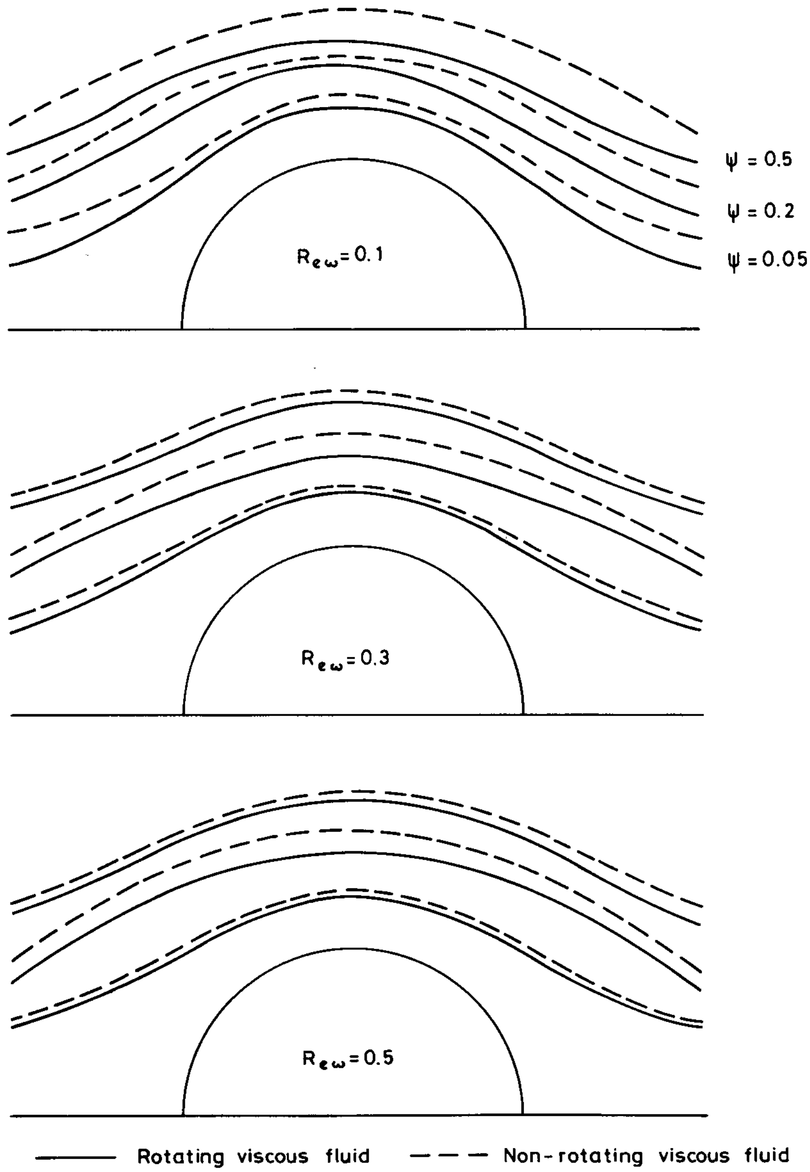

Figure 2. Stream lines for method of sweeps

The effect of rotation is to increase the drag, and as $R_{e\omega}$ increases, the drag increases.

The stream lines are plotted in Figure 2 for the method of sweeps for $R_{e\omega} = 0{\cdot}1$, 0·3, 0·5 and $\psi = 0{\cdot}05$, 0·2, 0·5. The dashed lines in each figure give the Stokes stream lines for the same values of ψ. The magnitude of the vorticity vector for $z = 0{\cdot}2$ and $R_{e\omega} = 0{\cdot}1$, 0·3 and 0·5 is plotted in Figure 3. The dashed line in Figure 3 give the vorticity for the non-rotating case at $z = 0{\cdot}2$. The effect of increasing $R_{e\omega}$ is to increase the magnitude of the vorticity vector at $z = 0{\cdot}2$ for the rotating case.

Table IV. Stokes stream function

z/θ	18	54	90	126	162
0·2	0·003301	0·022620	0·034543	0·022581	0·003277
0·4	0·015432	0·105747	0·161482	0·103362	0·015317
0·6	0·041156	0·282027	0·430672	0·281533	0·040851
0·8	0·087888	0·602263	0·919692	0·601208	0·087237
1·0	0·167023	0·604246	1·747784	0·142536	1·165785
1·8	1·319063	9·039041	13·803165	9·023210	1·309291

Vorticity (Stokes flow) at $z = 0{\cdot}2$

18°	36°	54°	72°	90°	109°	126°	144°	162°
0·117360	0·424578	0·804221	1·111169	1·228093	1·110298	0·802813	0·423169	0·116490

Table V.

$R_{e\omega}$	Drag
0·05	1·163709 D_s
0·1	1·167890 D_s
0·24	1·211938 D_s
0·3	1·22692 D_s

$D_s = 6\pi\mu aU$.

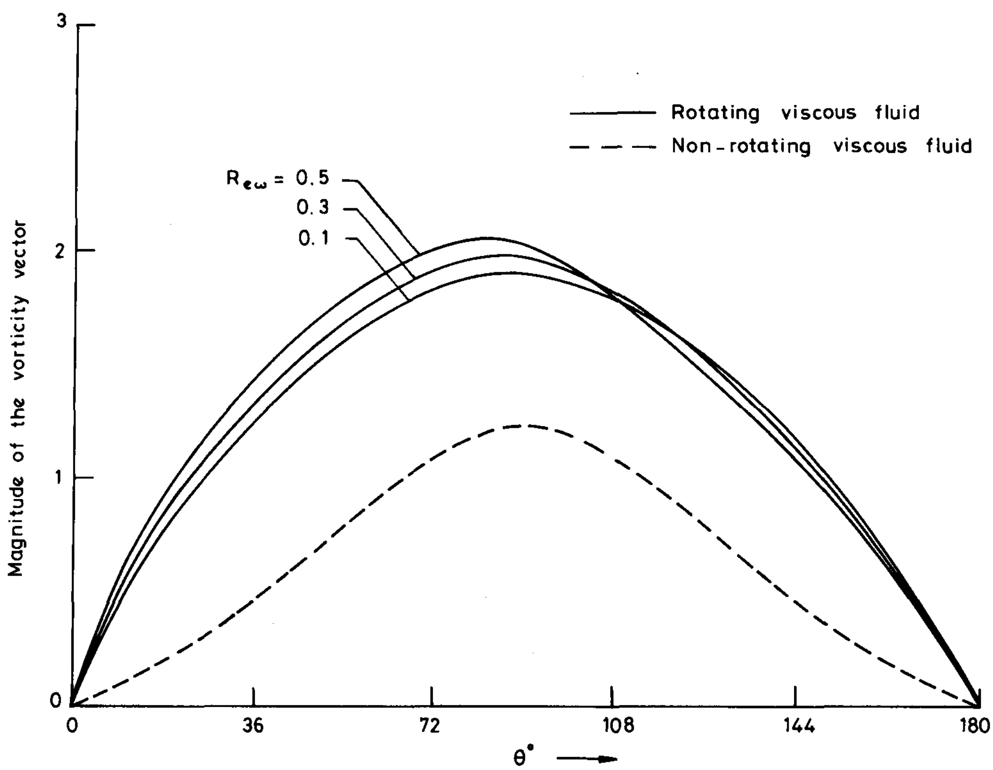

Figure 3. Magnitude of the vorticity vector for $z = 0{\cdot}2$

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